Section 6.1 Part 1: Discrete Random Variables

Today, you will learn how to:
- Compute probabilities using the probability distribution of a discrete random variable
- Calculate and interpret the mean (expected value) of a discrete random variable

How Many Children Are in Your Family?
Count up the number of children in your family (including yourself and any younger siblings or siblings over 18 years old). Be sure to include all stepbrothers/stepsisters and half-brothers/half-sisters.

1. Define the random variable $X$: Let $X = \text{number of children in family}$

<table>
<thead>
<tr>
<th>$X$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>$\frac{3}{13}$</td>
<td>$\frac{8}{13}$</td>
<td>$\frac{1}{13}$</td>
<td>$\frac{1}{13}$</td>
<td>$\frac{0}{13}$</td>
<td>$\frac{0}{13}$</td>
</tr>
</tbody>
</table>

2. Is this a valid probability model? Explain.
Yes! Each probability is between 0 and 1 and the sum of all prob = 1

No, we cannot have 0.7167 of a child.

4. Make a histogram to display the information with $X$ on the horizontal axis, and describe its shape.

5. Describe what $P(X \geq 3)$ means and find $P(X \geq 3)$.
$P(X \geq 3)$ is the probability of a randomly selected student having 3 or more children in their family.

$P(X \geq 3) = \frac{1}{13} + \frac{1}{13} + \frac{0}{13} + \frac{0}{13} = \frac{2}{13} \approx 15.4\%$

6. Describe what $P(X > 3)$ means and find $P(X > 3)$.
$P(X > 3)$ is the probability of a randomly selected student having more than 3 children in their family.

$P(X > 3) = \frac{1}{13} + \frac{0}{13} + \frac{0}{13} = \frac{1}{13} \approx 7.7\%$

7. Find the average of the $X$ values 1, 2, 3, 4, 5, 6. Does this value tell us the average number of children in the families of students in this class? If yes, explain. If not, why not?

$\frac{1+2+3+4+5+6}{6} = \frac{21}{13} = 1.61$ siblings. This does not give us the correct avg. # of siblings because it doesn’t “weight” correctly.

$M = \sum x_i p_i = 1 \left(\frac{3}{13}\right) + 2 \left(\frac{8}{13}\right) + \ldots + 6 \left(\frac{0}{13}\right) = 2$

If many, many students are randomly selected, the average # of children
### Big Ideas from Section 6.1 Part 1: Discrete Random Variables

<table>
<thead>
<tr>
<th>Discrete Random Variable:</th>
<th>Histogram:</th>
</tr>
</thead>
<tbody>
<tr>
<td>takes on a <strong>fixed</strong> set of values with gaps between values (Remember to define every time: “Let ( X = _________ )”)</td>
<td>can be be written as decimals, or fractions between 0 &amp; 1  [ \text{values of } X ]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Continuous Random Variable:</th>
<th>Expected Value (Mean) of a Random Variable:</th>
</tr>
</thead>
<tbody>
<tr>
<td>can take on <strong>any</strong> value within an interval of values (Find area under distribution to determine probabilities)</td>
<td>[ M = \sum X_i P_i ] (Given on formula sheet)</td>
</tr>
</tbody>
</table>

\[ M = \text{long term avg} \text{ (many, many...)} \]
Indiana University Bloomington posts the grade distributions for its courses online. Suppose we choose a student at random from a recent semester of this university’s Business Statistics course. The student’s grade on a 4-point scale (with A = 4) is a random variable $X$ with this probability distribution.

Let $X =$ student’s grade on 4-point scale

1. Write the event “the student got a C” using probability notation. Then find the probability. Show work!

   \[
P(\text{student got C}) = P(X = 2) = 1 - (0.011 + 0.032 + 0.362 + 0.457)
   \]

   * There is a 13.8% chance the student got a C.

2. Explain in words what $P(X \geq 3)$ means. What is this probability?

   $P(X \geq 3)$ is the probability the student gets a B or better.

   \[
P(X \geq 3) = 0.362 + 0.457 = 0.819
   \]

   “There is an...”

3. Make a histogram of this probability distribution. Describe its shape.

4. Calculate and interpret the expected value of $X$.

   \[
   E(X) = \mu = \sum x_i p_i = 0(0.011) + 1(0.032) + 2(0.138) + 3(0.362) + 4(0.457)
   \]

   \[
   E(X) = \mu = 3.222 \text{ points}
   \]

   If many, many students are chosen at random, the average grade is about 3.222 points.
Section 6.1 Part 2: Standard Deviation of Discrete Random Variables and Continuous Random Variables

Today, you will learn how to:
- Calculate and interpret the standard deviation of a discrete random variable
- Calculate the probability of an event using the probability distribution of a continuous random variable

What’s Your Wage?

Suppose you got a new job and each day your boss (Ms. Voinean) draws a slip of paper from a bag to determine your wage for the day. Let the random variable $X$ = daily wage ($\$ per hour$).

1. What is your wage for the day? $1$ Add your data to the table on the board and complete below:

<table>
<thead>
<tr>
<th>$X$</th>
<th>$$1$</th>
<th>$$5$</th>
<th>$$7$</th>
<th>$$10$</th>
<th>$$15$</th>
<th>$$25$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>$\frac{3}{16}$</td>
<td>$\frac{3}{16}$</td>
<td>$\frac{5}{16}$</td>
<td>$\frac{3}{16}$</td>
<td>$\frac{2}{16}$</td>
<td>$\frac{1}{16}$</td>
</tr>
</tbody>
</table>

2. Calculate and interpret the expected value of $X$.

$$E(X) = \mu = \sum X_i \cdot P_i = 1\left(\frac{3}{16}\right) + 5\left(\frac{3}{16}\right) + 7\left(\frac{5}{16}\right) + 10\left(\frac{3}{16}\right) + 15\left(\frac{2}{16}\right) + 25\left(\frac{1}{16}\right)$$

$$E(X) = \mu = $ 8.56 per hour

"If we draw many, many wages the expected average wage is about $8.56 per hour.

3. Recall from Chapter 1 that standard deviation tells us the typical distance from the mean. Complete the table to calculate the standard deviation for the probability distribution of daily wages.

4. Interpret the standard deviation.

"The wages typically vary by $5.72 per hour from the mean wage of $8.56 per hour.

5. Ms. Voinean decides that she would rather assign wages so that employees could get any amount from $10$ to $20$ and all are equally likely. Draw a graph to represent this probability distribution.

Uniform Distribution

$A = \frac{1}{X}$

$A = L \cdot w$

so $w = \frac{A}{L} = \frac{1}{10}$

$w = 0.1$

$w = \frac{1}{10}$

Uniform Distribution

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$w = 0.1$

$w = \frac{1}{10}$

Uniform Distribution
### Big Ideas from Section 6.1 Part 2: Continuous Random Variables

**Standard Deviation of a Discrete Probability Distribution:**
- Variance: \( \theta^2 = \sum (x_i - \mu)^2 p_i \) (Given on formula sheet)
- Standard Dev. = \( \theta \) (Not given)
- \( \theta_x = \sqrt{\sum (x_i - \mu)^2 p_i} \)  
  "Typical distance from \( \mu \)"

**Probability for Continuous Random Variables:**

1. **Just find area under curve!**

2. **Uniform vs. Normal Distributions**
- \( Z = \frac{X - \mu}{\theta} \)

#### Check Your Understanding #1 - 4

The heights of young women can be modeled by a Normal distribution with mean \( \mu = 64 \) inches and standard deviation \( \theta = 3 \) inches.
The heights of young women can be modeled by a Normal distribution with mean $\mu = 64$ inches and standard deviation $\sigma = 2.7$ inches. Suppose we choose a young woman at random and let $Y$ = her height (in inches).

1. What type of variable is $Y$ (discrete or continuous)? Explain.

   $Y$ is a continuous random variable because all heights are possible.

2. Interpret the standard deviation.

   $\sigma = 2.7$ inches
   The heights of young women typically vary by 2.7 inches from the average height of 64 inches.

3. Find $P(Y \leq 63)$. Interpret this value. Be sure to draw a diagram and show all formulas and steps.

   $\frac{Z}{\sigma_y} = \frac{63 - 64}{2.7} = -0.37$

   According to Table A, the area to the left of $z = -0.37$ is 0.3557.

   "There is a 0.3557 probability that a randomly selected female is 63 inches tall or shorter than 63 inches."

4. Find $P(68 \leq Y \leq 70)$. Interpret this value. Be sure to draw a diagram and show all formulas and steps.

   $\frac{Z_{68}}{\sigma_y} = \frac{68 - 64}{2.7} = 2.22$
   $\frac{Z_{70}}{\sigma_y} = \frac{70 - 64}{2.7} = 1.48$

   $P(68 < Z < 70) = 0.9868 - 0.9366 = 0.0502$

   "There is a 0.0502 probability that a randomly selected female is between 68 to 70 inches tall."